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# Bayes Tutorial using R and JAGS


James Brownlow

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Tutorial on how to obtain, load and use fundamental Bayesian analysis for flight test: examples include estimation of a binomial proportion, parameters of a normal distribution and reliability – homogeneous Poisson process trend estimation and prediction

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# Bayes using R and JAGS

## 12-14 May, 2015

**James Brownlow**
**812 TSS/ENT**
**661 277-4843**

U.S. AIR FORCE

# Overview

- Introduction
- Background
- Uncertainty Analysis
- Systematic Error
- Random Error
- Conclusion

# Goals

- Load R, JAGS onto your laptop! (Disk set up for Windows)
- Learn the fundamentals of Bayesian analyses
- Learn how to run Bayesian analyses from within R, using JAGS, and interpret the results
- Learn how to evaluate “goodness of fit” for a Bayes model
- Learn predictive posterior distributions, hierarchical modeling

# background

- What is Bayesian data analysis?  Why Bayes?
- Why R and Bugs
- Bayesian examples:
  - binomial,
  - normal distribution
  - reliability applications
- Model checking
- Bayes estimate and prediction of lambda in HPP reliability analysis

# What are BUGS and R?

- Bugs – Bayesian analysis Using Gibbs Samplers
  - BUGS is a language used to set up Bayesian inference
  - JAGS (Just Another Gibbs Sampler) is the Bayes software that runs within R
- R – GNU statistical analysis package
  - Open source language for statistical computing and graphics
  - Well vetted, used in virtually every university on this planet
- Bugs from within R
  - Offers flexibility in data manipulation before the analysis and display of inferences after
  - Avoids tedious issues of working with Bugs directly

# A brief prehistory of Bayesian data analysis

- Reverend Thomas Bayes (1763)
  - Links statistics to probability
- Laplace (1800)
  - Normal distribution
  - Many applications, including census [sampling models]
- Gauss (1800)
  - Least squares
  - Applications to astronomy [measurement error models]
- Keynes, von Neumann, Savage (1920’s-1950’s)
  - Link Bayesian statistics to decision theory
- Applied statisticians (1950’s-1970’s)
  - Hierarchical linear models
  - Applications to medical trials, conjugate priors
  - 1990s MCMC techniques, increased computing power

# A brief history of Bayesian data analysis, BUGS, and R

- "Empirical Bayes" (1950's-1970's)
  - Estimate prior distributions from data
- Hierarchical Bayes (from 1970)
  - Include hyper parameters as part of the full model
- Markov chain simulation (from 1940's [physics] and 1980's [statistics])
  - Computation with general probability models
  - Iterative algorithms that give posterior simulations (not point estimates)
- R code (open source) for statistical applications (1994)
- lme() and lmer() functions by Doug Bates for fitting hierarchical linear and generalized linear models

- Bugs (from 1994)
  - Bayesian inference Using Gibbs Sampling. Developed explicitly for Bayesian statistics

# What is Bayesian data analysis? Why Bayes?

- Effective and flexible
- Combine information from different sources
- Examples of previous uses of Bayes from flight test include:
  - Radar systems analysis
  - Regression testing (“same as old”)
  - Reliability applications
  - Multilevel regression, hierarchical modeling- test unit parameters are not all the same, but are drawn from “parent” distribution

# Structure of the tutorial

- Computer use
- Example code included R, and JAGS
- “Follow along” computer demonstrations
- Feel free to Interrupt with questions
- Preliminaries include
  - How to set up a BUGS model in R
  - Use R to facilitate posterior distribution inference and diagnostics

# Structure.. continued

- Understanding how BUGS works and basic requirements for using JAGS with BUGS
- Examples – Use Bayesian approach to
  - Estimate the parameter of a binomial distribution
  - Estimate parameters of a log-normal distribution
  - Do reliability analysis- examine trend in an assumed HPP

# What are BUGS and R?

- BUGS (Bayesian Inference Using Gibbs Sampling)
  - Represent/Fit Bayesian statistical models
  - Is a “language” designed to express Bayesian models
- R
  - Open source language for statistical computing and graphics
- BUGS from within R
  - Run MCMC based Bayesian analyses from within R
  - Offers flexibility in data manipulation before the analysis and display of inferences after
  - Avoids tedious issues of working with Bugs directly
- [Open R:  binomial]

# Bayes, Bugs, and R

- Use R for data manipulations and various analysis models
- Use BUGS within R to fit complex Bayesian models
- User R to summarize results:
  - Statistical inference from a posterior distribution
  - check that fitted model makes sense (validity of the BUGS) result
  - check for validity of model implemented in BUGS

# Fitting a Bayesian model in R and Bugs… We’ll cover

- What’s required for a BUGS model
- Setting up data and initial values in R
- Running BUGS and checking results (convergence, model adequacy)
- Displaying the posterior distribution, draw inferences

# EVERY R-script using JAGS looks like

1. Clear the workspace, get R2jags

   rm(list=ls())

   require(R2jags) #interface: R and JAGS

2. Enter the "BUGS" model using R-function cat() …. As shown on next slide

# Class Example: Estimate the probability of success of a rocket launch for companies with limited launch/design experience

- Example is from Hamada et al., *Bayesian Reliability*, Springer, 2008
- Data: 11 companies with little launch/design experience. Objective is to develop a statistical model to predict launch success of a “new” company
- Model as a Bernoulli process- rocket launch was a success or it was not

# Example: here is historical data (1980-2000)

| Vehicle | Outcome | Coded… |
|---|---|---|
| Pegasus | Success | 1 |
| Percheron | Failure | 0 |
| AMROC | Failure | 0 |
| Conestoga | Failure | 0 |
| Ariane 1 | Success | 1 |
| India SLV-3 | Failure | 0 |
| India ASLV | Failure | 0 |
| India PSLV | Failure | 0 |
| Shavit | Success | 1 |
| Taepodong | Failure | 0 |
| Brazil VLS | Failure | 0 |

# Begin with Maximum Likelihood Estimation of p

- Probability of success is p, failure is (1-p)
- $f(y|n,p) = \binom{n}{y} p^y (1-p)^{n-y}$
- Log-likelihood: log[f(y|n,p)] α y * log(p) + (n-y) * log(1-p)
- y = 3, n=11, take first derivative of log-likelihood, set =0,
- 0 = d( log(f(y|n,p))/d(p) = y/p - (n-y)/(1-p), solve for p
- p = 3/11 = 0.272

# Enter the BUGS model

- cat(‘ # model is a character string

```
model {
    for(i in 1:n) {
        x[i] ~ dbern(theta)
    }
    theta ~ dbeta(1,1)  #prior on theta
    }’ , # end of BUGS model
file=“fileName.txt”)  # end of cat()
```

# R and Bugs for classical inference

- Estimate the parameter of a binomial distribution using R / BUGS
- Displaying the results in R or rmarkdown
- Use two priors for the analysis
  - “vague” prior- uniform across (0,1)
  - “informative” prior- p  around 0.3

# Required to run jags: data

```
fileNameData=list(x=c(rep(1,3), rep(0,8)),
   n=11)
```

- NOTE data must be a list()

# Required to run jags: inits

```
fileNameInits = function() {
  list(theta = rbeta(1,1,1))}
```

- NOTE inits must be a function, return a list (allows for multiple MCMC chains)
- Inits can be a NULL function- i.e. let JAGS pick initial values of parameters

# Required to run jags: parameters to save

fileNameParms = c(“theta”)

- NOTE: parameters must be a text “collection” (vector) of  variable names

# Summary, so far…

1. data must be a list
2. Inits must be a function
3. parameters must be a vector of text name(s) of the variable(s) we want to examine, use for inference

# Run jags

- Call to jags: (from within an R script)

```
fileNameJags=jags(
        data=fileNameData,
        inits = fileNameInits,
        parameters.to.save = fileNameParms,
        model.file="fileName.txt",
            .
            .
```

# Run jags continued

```
n.iter = 2000,
n.thin = 1,
n.burnin = 1000,
n.chains = 4,
DIC = TRUE)
```

- Notice it's all case sensitive!

# Put together….

```
fileNameJags=jags(
        data = fileNameData,
        inits = fileNameInits,
        parameters.to.save = fileNameParms,
        model.file = “fileName.txt”,
        n.iter = 2000,
        n.thin = 1,
        n.burnin = 500,
        n.chains = 4,
        DIC = TRUE)
```

# To get some diagnostics, and a plot:

- fileNameJagsMC2 = autojags(fileNameJags)
- attach.jags(fileNameJagsMC2)
- plot(density(theta))

# Now you try it! (exercise1.R)

- Exercise: set up and run the binomial distribution- estimate theta, get a posterior density function of theta
  - Use 3 successes in 11 trials
  - Uniform prior distribution on p
  - Parameter θ, plot posterior of θ
  - Repeat using a beta distribution for prior p, parameters (alpha=2.24, beta=2)

# Overview of Bayesian data analysis

- Decision analysis for reliability
- Where did the “prior distribution” come from?
- Simulation-based model checking

# Result dependent on prior!

- Different priors yielded different results!
- One can incorporate prior information into analyses
- Prior distributions may be useful:
  - Suppose we do a reliability test and have no failures in 311 hours – what can we say about MTBF?

# Decision analysis for reliability

- Bayesian inference
  - Prior(θ) + data + likelihood(data|θ) = posterior(θ)
  - Where did the prior distribution come from?

# Prior distribution

- Example of Bayesian data analysis
- Binomial
  - Assume a beta prior for p
  - Incorporate data to update estimate of p, MTBF
  - On the disk- binomial.R
- HPP model
  - Number of failures proportional to interval length
  - Poisson model
  - On the disk– poisson.R
- In both cases: model is flexible-
  - add arbitrary time intervals, new data

# More on Bayesian inference

- Allows estimation of an arbitrary number of parameters
- Summarizes uncertainties using probability
- Combines data sources
- Model is testable

# OK, let's, estimate p(successful launch) using Bayes..

- MLE has excellent "large sample" properties, but, not so good for small to medium samples:
  - large sample properties of MLE do not pertain to complicated applications
  - MLE is not appropriate for hierarchical models
  - MLE does not work well when parameters are close to boundary of the parameter space
  - Deriving analytic expressions is difficult in high-dimension situations
- All of these difficulties are, of course, eliminated in Bayesian estimation

# Fundamentals of Bayesian Inference

- Frequentist estimation includes a confidence interval- i.e. an interval that will contain the true value of the parameter  some specified proportion of the time in an infinite sequence of repetitions of the experiment
- Bayesian estimation combines knowledge of the parameter available before sample data are analyzed with information gathered during an experiment
  - Update the estimate of the parameter
  - Summarize knowledge of the parameter using a probability density function

# Bayes fundamentals – the mechanism

$$p(\theta \mid y) = \frac{f(y \mid \theta)\, p(\theta)}{m(y)}$$

$$m(y) = \int f(y \mid \theta)\, p(\theta)\, d\theta$$

$p(\theta|y)$ is the posterior density of $\theta$
$p(\theta)$ is the prior density of $\theta$
$m(y)$ is the marginal density of the data, and
$f(y|\theta)$ is the sampling density of the data

# And the parameter of interest, θ

$$E(\theta) = \int \theta \, f(\theta \mid y) d\theta$$

Once we get $f(\theta|y)$ we can estimate any density-related parameter!

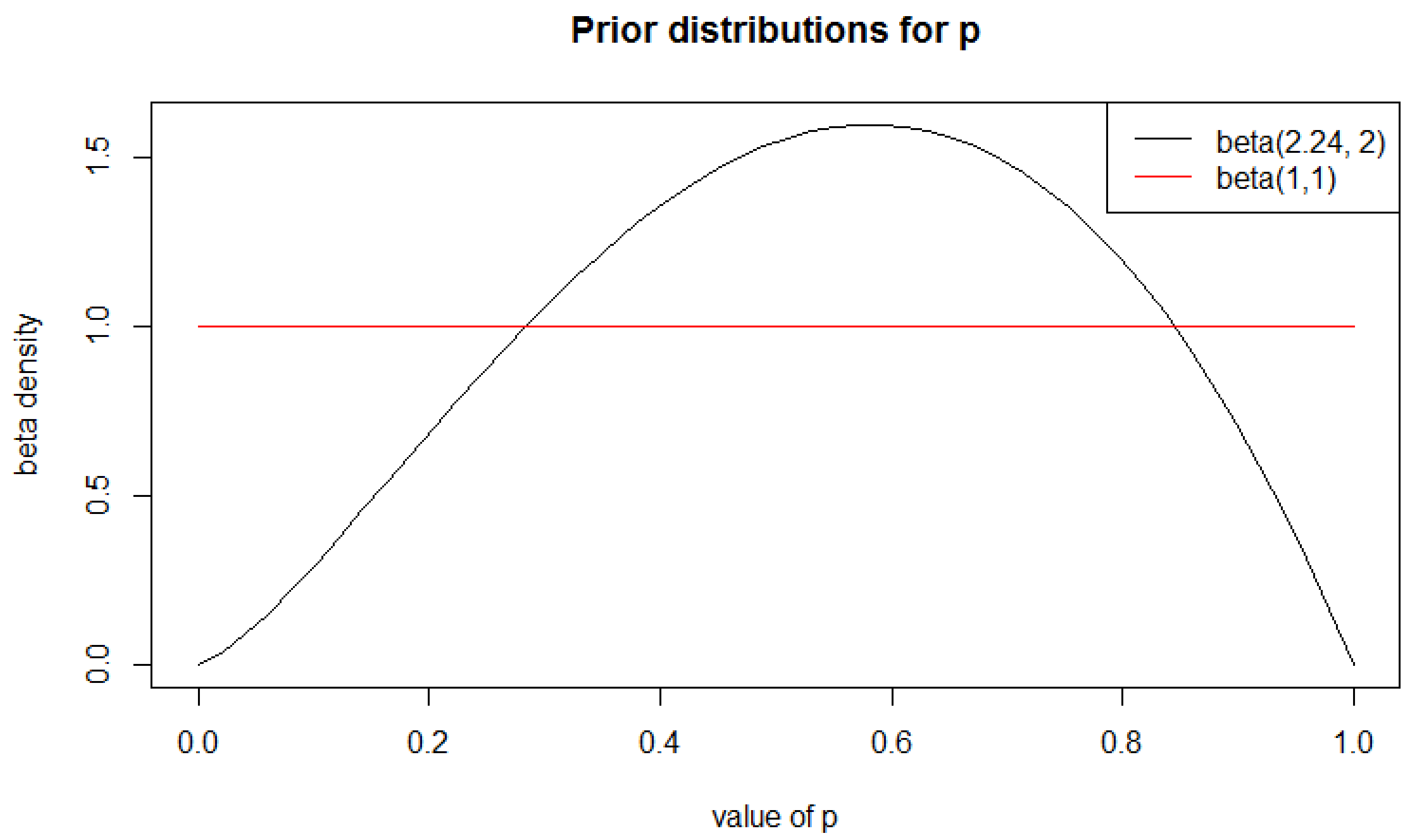
# The prior distribution

- In the launch vehicle example, θ is the parameter of interest, the probability of success of a launch
- Prior information:
  - Diffuse: θ can be anywhere in the interval (0,1)
  - Informative: more specific information about θ may be available- past history indicates that θ is concentrated near 0.4
- We will look at the launch problem using first the “diffuse” (aka vague) prior and then the “informative” prior

# Priors

- *A priori* we take all values in the interval (0,1) to be equally likely for θ: $p(\theta) = 1, \quad 0 < \theta < 1$
- OR we use previous experience with launch vehicles to assert that the probably of a successful launch is around 0.55, and choose for the prior a beta distribution with parameters $\alpha=2.4$, and $\beta = 2$
  - Mean of the beta distribution is $\alpha/(\alpha+\beta) = 0.545$, and
  - Median of the beta distribution is $(\alpha-1)/(\alpha+\beta-1) = 0.583$

# Beta prior for p



# Likelihood function

- Likelihood = bernouli, so result is either 1, or 0 with probability p, repeated 11 times
- Can use a single likelihood, binomial- three successes in 11 trials

# Now let's use Bayes rule to estimate posterior distributions of the parameter, θ

- Bayes Rule: posterior α likelihood * prior
- Implement this in the "BUGS" language
- Call "jags" to develop the estimate of the posterior distribution, $f(\theta \mid y)$

# EVERY R-script to use JAGS does the following

- Clear the workspace, get R2jags
  rm(list=ls())
  require(R2jags)

- Enter the “BUGS” model using R-function “cat()” …. As shown on next slide

# Enter the BUGS model

- cat(‘

```
model {
    for(i in 1:n) {
    x[i] ~ dbern(theta)
}
theta ~ dunif(1,1)  #prior on theta
}’ , # end of BUGS model
file=“fileName.txt”)  # end of cat()
```

# Required to run jags: data

- fileNameData = list(x=c(1,0,0,0,1,0,0,0,1,0,0), n=11)

- NOTE data must be a list()

# Required to run jags: inits

- fileNameInits = function() {

```
                    list(theta = runif(1,0,1))
        }
```

- NOTE “fileNameInits” must be a function, return a list (allows for multiple MCMC chains)

# Required to run jags: parameters to save

- fileNameParms = c("theta")

- NOTE: fileNameParms must be a text collection of one or more variable names

# Quick Check: need to input data, inits, and parameters to save

1. data must be a list
2. Inits must be a function
3. parameters must be a collection of text, naming variables we want to examine

# Run jags

- Call to jags:
- fileNameJags=jags(

  data=fileNameData,

  inits = fileNameInits,

  parameters.to.save = fileNameParms,

  model.file="fileName.txt",

  .

  .

# Run jags continued

n.iter = 2000,

n.thin = 1,

n.burnin = 1000,

n.chains = 4,

DIC = TRUE)

- Notice it’s all case sensitive!

# Put together….

- fileNameJags=jags(

```
data = fileNameData,
inits = fileNameInits,
parameters.to.save = fileNameParms,
model.file = “fileName.txt”,
n.iter = 2000,
n.thin = 1,
n.burnin = 1000,
n.chains = 4,
DIC = TRUE)
```

# Get some diagnostics, and a plot

- fileNameJagsMC2 = autojags(fileNameJags)

- attach.jags(fileNameJagsMC2)

- plot(density(theta))

# Now you try it!

- Exercise: set up and run the binomial distribution- estimate theta, get a posterior density function of theta
  - Use 25 successes in 289 trials
  - Uniform prior distribution on p
  - Parameter θ, plot posterior of θ

# Decision analysis for reliability

- Bayesian inference
  - Prior(θ) + data + likelihood(data|θ) = posterior(θ)
  - Where did the prior distribution come from?

# Prior distribution

- Example of Bayesian data analysis
- HPP model
  - Number of failures proportional to interval length
  - Poisson model
  - On the disk– poisson.R
- Data model
  - Flexible:  arbitrary time intervals,
  - Add data as it is acquired

# Types of prior distributions

- Two traditional extremes:
  - Non-informative priors
  - Subjective priors
- Problems with each approach
- New idea:  weakly informative priors
- Illustration with a logistic regression example

# Bayesian inference- reliability

- Set up and compute model
  - Use data at hand; update as more data becomes available
  - Inference using iterative simulation (Gibbs sampler)
- Inference for quantities of interest
  - Uncertainty distribution for mean time between failures
- Model checking
  - Do inferences make sense?
  - Compare replicated to actual data, cross-validation
  - Dispersed model validation (“beta-testing”)
- Set up model checking in the HPP program

# Bayesian inference – summary, so far

- Set up and compute model
  - Use data at hand; update as more data becomes available
  - Inference using iterative simulation (Gibbs sampler)
- Inference for quantities of interest
  - Uncertainty dist for mean time between failures
- Model checking
  - Do inferences make sense?
  - Compare replicated to actual data, cross-validation
  - Dispersed model validation (“beta-testing”)
- Set up model checking in the HPP program

# Bayesian inference

- Allows estimation of an arbitrary number of parameters
- Summarizes uncertainties using probability
- Combines data sources
- Model is testable (falsifiable)

# Model checking

- Basic idea:
  - Display observed data (always a good idea anyway)
  - Simulate several replicated datasets from the estimated model
  - Display the replicated datasets and compare to the observed data
  - Comparison can be graphical or numerical
- Generalization of classical methods:
  - Hypothesis testing
  - Exploratory data analysis
- Crucial “safety valve” in Bayesian data analysis

# Model checking and model comparison

- Generalizing classical methods
  - t tests
  - chi-squared tests
  - F-tests
  - $R^2$, deviance, AIC
- Use estimation rather than testing where possible
- Posterior predictive checks of model fit
- DIC for predictive model comparison

# Model checking: posterior predictive tests

- Test statistic, "T(y)"
- Replicated datasets y.rep(k), k=1,…,n.sim
- Compare T(y) to the posterior predictive distribution of T(y.rep(k))
- Discrepancy measure T(y,theta(k))
  - Look at n.sim values of the difference, $T(y,theta^k) - T(y.rep^k,theta^k)$
  - Compare this distribution to 0

# Model comparison: DIC (deviance information criterion)

- Generalization of “deviance” in classical GLM
- DIC is estimated error of out-of-sample predictions
- DIC = posterior mean of deviance
- Compare the two binomial models:
  - uniform prior (non-informative) and
  - beta(2.4, 2) prior (informative)

# Understanding the Gibbs sampler and Metropolis algorithm

- Monitoring convergence
- examples of good and bad convergence
- n.chains: at least 2, preferably 4
- Role of starting points
- R-hat
  - Less than 1.05 is good
- Effective sample size
  - At least 100 is good

# Concluding discussion

- What should you be able to do?
  - Set up hierarchical models in Bugs
  - Fit them and display/understand the results using R
  - Compare to estimates from simpler models
  - Use Bugs flexibly to explore models
- What questions do you have?

# Software resources

- **Bugs**
  - User manual (in Help menu)
  - Examples volume 1 and 2 (in Help menu)
  - Webpage (http://www.mrc-bsu.cam.ac.uk/bugs) has pointers to many more examples and applications
- **R**
  - ?command for quick help from the console
  - Html help (in Help menu) has search function
  - Complete manuals (in Help menu)
  - Webpage (http://www.r-project.org) has pointers to more
- Appendix C from “Bayesian Data Analysis,” 2nd edition, has more examples of Bugs and R programming for the 8-schools example
- “Data Analysis Using Regression and Multilevel/Hierarchical Models” has lots of examples of Bugs and R.

# References

***General books on Bayesian data analysis:***

Bayesian Data Analysis, 2nd ed., *Gelman, Carlin, Stern, Rubin (2004)*
Bayesian Reliability, *Hamada, Wilson, Reese, Martz (2008)*

***General books on multilevel modeling***

Data Analysis Using Multilevel/Hierarchical Models, *Gelman and Hill (2007)*
Hierarchical Linear Models, *Bryk and Raudenbush (2001)*
Multilevel Analysis, *Snijders and Bosker (1999)*

***Books on R***

An R and S Plus Companion to Applied Regression, *Fox (2002)*
An Introduction to R, *Venables and Smith (2002)*